WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS METHODS
Calculus, trigonometry and DRV's
Test 3

Name: Solutions

Marks: /45

Calculator Free (20 marks)

Time allowed: 50 mins

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1. [2 marks]

Determine if each of the $p(x)$ as described are discrete probability functions. Justify your answer in either case.

a)

x	0	1	2	5
$P(X=x)$	-0.1	0.1	0.4	0.6

No, negative probability of -0.1 ie $P(0)=-0.1$

[1]

b)

x	-3	-2	1	4
$P(X=x)$	0.1	0.3	0.2	0.4

Yes, probabilities sum to 1.

[1]

2. [3 marks]

Given a binomial variable has a mean of 12 and a standard deviation of $\sqrt{8}$, find p, the probability of success and n, the number of trials.

$np = 12$ and $np(1-p) = 8$

ie $12(1-p) = 8$ (1)

$1-p = \frac{2}{3}$

$\therefore p = \frac{1}{3}$ (1)

$\therefore n = \frac{12}{\left(\frac{1}{3}\right)} = 36$ (1)

5

3. **[10 marks]**

Determine:

a) $\frac{d}{dx}\cos^5(3x)$ $= 5\cos^4(3x)\,(-\sin(3x)\cdot 3)$

$= -15\sin(3x)\cos^4(3x)$

[2]

b) $\frac{d}{dx}e^{2x+1}\tan(5x)$ $= 2e^{2x+1}\tan(5x) + e^{2x+1}\frac{5}{\cos^2(5x)}$

$= e^{2x+1}\left(2\tan(5x) + \frac{5}{\cos^2(5x)}\right)$

[2]

c) $\int \frac{\sin(5x)}{4}dx$ $= \frac{1}{4}\left(\frac{-\cos(5x)}{5}\right)$

$= -\frac{1}{20}\cos(5x) + c$

[2]

d) $\int \cos(x)\sin^3(x)\,dx$ $= \frac{1}{4}\sin^4(x) + c$

[2]

e) $\frac{d}{dx}\int_e^{x^3}\cos(3t)\,dt$ $= \cos(3x^3)\cdot 3x^2$

$= 3x^2\cos(3x^3)$

[2]

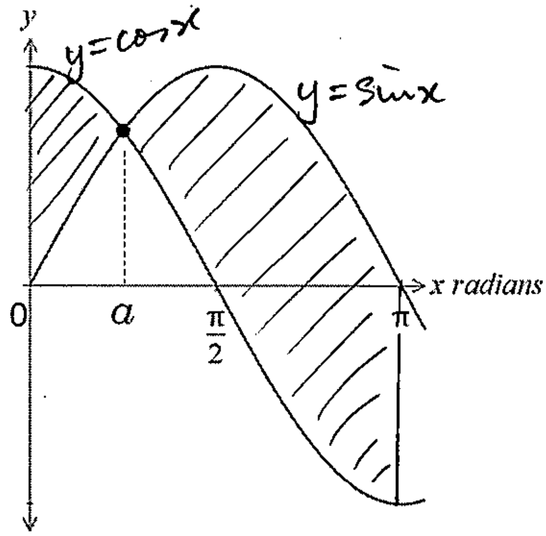
10

4. [5 marks]

Determine the area trapped between the functions $y=\sin(x)$, $y=\cos(x)$, $x=0$ and $x=\pi$.

Hint: First, determine a.

$\alpha = \pi/4$



(1) (1)

$$\text{Area} = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi} (\sin x - \cos x)\,dx$$

$$= \left[\sin x + \cos x\right]_0^{\pi/4} + \left[-\cos x - \sin x\right]_{\pi/4}^{\pi} \quad (1)$$

$$= \sin(\pi/4) + \cos\pi/4 - \sin(0) + \cos(0) - \left(\cos(\pi) + \sin(\pi) - \left(\cos(\pi/4) + \sin(\pi/4)\right)\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 - \left[-1 + 0 - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right] \quad (1)$$

$$= \sqrt{2} - 1 + 1 + \sqrt{2}$$

$$= 2\sqrt{2} \text{ sq units.} \quad (1)$$

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Calculator Section (25 marks)

5. [6 marks]

A company produces fruit sweets coated with either dark chocolate or milk chocolate.
A large number of these fruit sweets are placed in a box. Twenty percent of the sweets in the box are coated with dark chocolate.

a) A random sample of ten sweets is taken from the box, explain the meaning of the calculation ${}^{10}C_4\,(0.2)^4(0.8)^6$ with respect to this sample?

The probability of randomly selecting 4 dark chocolates from a sample of 10 sweets from the box. (1) (1)

[2]

b) (i) Find n given that ${}^{n}C_4\,(0.2)^0(0.8)^n = 0.16777$

ie $0.8^n = 0.16777$ (1)

Solving $n = 8$ (nearest integer) (1)

[2]

(ii) Explain the meaning of your answer from b) (i) with respect to the fruit sweets.

The probability of getting no dark chocolates when 8 sweets are choosen. (1) (1)

[2]

6

6. [8 marks]

The random variable X has probability distribution:

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

Given that $E(X) = 4.5$, determine:

a) The value of p and q.

$E(x) = 4.5$ $\qquad 0.55 + p + q = 1$

ie $\quad p + q = 0.45 \quad (1)$

$\Downarrow$

and $\quad 1(0.2) + 3p + 5(0.2) + 7q + 9(0.15) = 4.5$

$3p + 7q = 1.95 \quad (2)$

Solve (1) + (2) simultaneously

$p = 0.3$

$q = 0.15$

[3]

b) $P(4 < x \leq 7)$ $\qquad 0.2 + 0.15 = 0.35$

[1]

Given that $E(X^2) = 27.4$, determine:

c) $Var(X)$ $= \Sigma(x^2) - E(x)^2$

$= 27.4 - 4.5^2$

$= 7.15$

[2]

d) $E(19 - 4X)$ $= -4(4.5) + 19$

$= 1$

[1]

e) $Var(19 - 4X)$ $= 4^2(7.15)$

$= 114.4$

[1]

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7. **[3 marks]**

Suppose that 5% of all items coming off a production line are defective. Assume the manufacturer packages his items in boxes of six and guarantees "double your money back" if more than two items in a box are defective. On what percentage of the boxes will the manufacturer have to pay double money back?

$X \sim B(6, 0.05)$ (1)

$P(X > 2) = 0.0022$ (1)

[Binomial CDF (3, 6, 6, 0.05)]

∴ manufacturer will have to pay back double money 0.22% of the time.
(1)

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8. **[8 marks]**

A soldier fires shots at a target at distances ranging from 25 m to 90 m. The probability of him hitting the target with a single shot is p. When firing from a distance of d m, $p = \frac{3}{200}(90-d)$. Each shot is fired independently.

The soldier fires 10 shots from a distance of 40 m.

a) Determine the probability that:

(i) Exactly 6 shots hit the target.

$d = 40 \Rightarrow p = 0.75$ (1)

$X \sim B(10, 0.75)$ (1)

$P(X=6) = 0.146$ (1)

[3]

(ii) At least 8 shots hit the target.

$P(X \geq 8) = 0.5256$

(1) (1)

[2]

The soldier fires 20 shots from a distance of x m.

b) Determine to the nearest integer, the value of x if the soldier has an 80% chance of hitting the target *at least once*.

$X \sim B(20, p)$

$P(X \geq 1) = 0.8$

ie $1 - P(X=0) = 0.8$

$P(X=0) = 0.2$

ie $(1-p)^{20} = 0.2$ (1)

$\therefore$ $p = 0.0773$ (1)

So $0.0773 = \frac{3}{200}(90-x)$

$\therefore$ $x = 84.85$

[3]

$\therefore$ $x = 85$ m (1) (nearest int.)

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